Poster Abstract: Communication Reliability Analysis from Frequency Domain

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***Abstract*—There is an urgent demand on efficient methodologies to model and analyze the end-to-end delay distribution of networked systems since end-to-end delay beyond deadline may result in catastrophic consequences. Traditional methods based on time domain analysis, however, are not efficient as the network scale and the complexity increase. In this paper, we propose a novel theoretical framework to analyze the end-to-end delay distribution of large-scale networked systems from the frequency domain. We use a signal flow graph to model the delay distributions of a networked system and prove that the end-to-end delay distribution is indeed the inverse laplace transform of the transfer function of the signal flow graph. Two efficient methods, Cramers rule based method and Mason gain rule based method, are adopted to obtain the transfer function. Further, we propose an efficient method using the dominant poles of the transfer function to improve the end-to-end delay distribution. Theoretical analysis and extensive evaluations show the effectiveness of the proposed approach.**

I. Introduction

Communication reliability is an essential requirement for the dependability of mission-critical systems. For example, a single link failure, or communication delay of feedback signals may destroy the nuclear plant and kill hundreds of people. The end-to-end delay distribution can be used as a metric to measure communication reliability [1]. Traditional methods of end-to-end delay analysis based on time domain analysis are not convenient to be used here because they calculate the end-to-end delay distribution by convoluting the delay distributions of individual nodes and links, and the computational cost is extremely high when the network is complicated and its scale is large. In addition, in most situations, traditional methods can not get closed-form solutions. Moreover, previous work rarely provide methods for us to improve the end-to-end delay distribution efficiently.

In this paper, we propose a new theoretic framework for communication reliability (or end-to-end delay distribution) analysis and enhancement of networked systems through frequency control theory. We use a signal flow graph to model the delay distributions of a networked system, which does not depend on any certain network structure. The end-to-end delay distribution in the time domain is indeed the inverse laplace transform of the transfer function of the signal flow graph. Two methods, Cramer's rule based method and Mason Gain Rule based method, are adopted to efficiently obtain the transfer function. By analyzing the time response of the transfer function, we can obtain the end-to-end delay distribution. Lastly, dominant pole and its related theory are introduced to figure out the dominant links (or bottleneck links). By improving the delay distribution of dominant link, the end-to-end delay distribution of the networked systems can be efficiently improved.

TABLE I
PARAMETERS OF THE EXAMPLE NETWORKED SYSTEM

Links	Delay Distributions	Selected Probability
$e(1,2)$	$D(1,2) = \frac{9}{s+9}$	$p(1,2) = 0.7$
$e(1,4)$	$D(1,4) = \frac{7}{s+7}$	$p(1,4) = 0.3$
$e(2,3)$	$D(2,3) = \frac{5}{s+5}$	$p(2,3) = 1$
$e(3,6)$	$D(3,6) = \frac{1}{s+1}$	$p(3,6) = 1$
$e(4,3)$	$D(4,3) = \frac{4}{s+4}$	$p(4,3) = 0.2$
$e(4,5)$	$D(4,5) = \frac{7}{s+7}$	$p(4,5) = 0.8$
$e(5,2)$	$D(5,2) = \frac{0.5}{s+0.5}$	$p(5,2) = 0.3$
$e(5,4)$	$D(5,4) = \frac{5}{s+5}$	$p(5,4) = 0.1$
$e(5,6)$	$D(5,6) = \frac{8}{s+8}$	$p(5,6) = 0.6$

II. System Modeling

We use signal flow graph [2] to model the delay distributions of a networked system. For example, Figure 1 shows an example signal flow graph model corresponding to delay of a networked system. The parameters of the example system are listed in Table I.

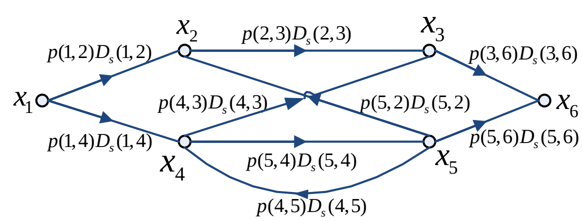

Fig. 1. An example signal flow graph corresponding to delay of a networked system.

The transfer function of the signal flow graph G is defined as T_s. We prove that: *The end-to-end delay distribution of a system is equal to the inverse laplace transform of the transfer function of its corresponding signal flow graph G. That is,* $d_{e2e} = \mathcal{L}^{-1}[T_s]$ Therefore, we convert the end-to-end delay distribution calculation problem into the problem of obtaining the transfer function of the corresponding signal

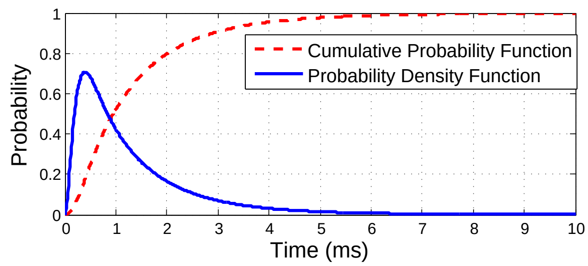

Fig. 2. Probability density function and cumulative distribution function of e2e delay.

flow graph. Further, we have: *The impulse response of the transfer function equals the probability density function of the end-to-end delay distribution*, and *the step response equals the cumulative distribution function of the end-to-end delay distribution*.

III. Delay Distribution Analysis

Cramer's Rule based method and Mason Gain Rule based method are widely used in control theory [2] to calculate the system's transfer function. We take the system in Figure 1 as an example. The system transfer function can be calculated by either of the two methods, which is:

$$T_s = \frac{N(s)}{D(s)}, \quad (1)$$

where

$$N(s) = 89.628(s+8.958)(s+3.782)(s+3.515)(s+0.5522)(s+5.1701+0.8974i)(s+5.1701-0.8974i), \quad (2)$$

$$D(s) = (s+9)(s+8)(s+7.949)(s+7)(s+5)(s+4.051)(s+4)(s+1)(s+0.5). \quad (3)$$

Figure 2 shows the impulse response and the step response of our example system in Figure 1. The impulse response curve describes the relative likelihood for message delivered at a given time. It will move to the left when the mean end-to-end delay is shorter, and be much sharper when jitter (or delay variation) is smaller. We find that the end-to-end delay mainly occurs between 0 and 2 seconds. The probability of the end-to-end delay larger than 5 seconds is very small.

IV. End-to-end Delay Distribution Improvement

For the system transfer function T_s, the zeros of this function, $s = z_h$, are those values of s for which $T_s(z_h) = 0$. The poles of this function, $s = p_k$, are those values of s for which $|T_s(b_k)| = \infty$. There exists dominant poles [2] which determine the system time response (i.e., end-to-end delay distribution). The dominant poles are usually close to the imaginary axis, while the non-dominant poles are usually far away from the left of the dominant poles or be near zero when not far to the left of the dominant poles. Since the system is composed by links, the poles of the system depend on the delay distributions of individual links. Thus, there exist some links named *dominant links* in this paper which decide the dominant poles of the system. By improving the delay distribution of dominant link, the end-to-end delay distribution of the networked systems can be efficiently improved.

Take the networked system shown in Figure 1 as an example. We find that $(-1, 0)$ is the dominant pole of T_s. The reasons are: Firstly, although the pole $(-0.5, 0)$ is not to the left of pole $(-1, 0)$, there exists a zero $(-0.5522, 0)$ close to it; Secondly, the other poles are far to the left of $(-1, 0)$. From Table I, we find that the transfer function of link $e(3, 6)$ is $T(3, 6) = D(3, 6)p(3, 6) = \frac{1}{s+1}$, so its pole is $(-1, 0)$ which is the same as the dominant pole of T_s. Thus, we judge that link $e(3, 6)$ is the dominant link of the networked system. Thus, by improving $e(3, 6)$, we can improve the the end-to-end delay significantly.

In order to evaluate it, we improve $e(3, 6)$ and $e(5, 2)$ (the worst link) to $\frac{2}{s+2}$ respectively and get the corresponding the end-to-end delay distribution as shown in Figure 3. Obviously, when we improve $e(3, 6)$, probability density function curve is sharper, and cumulative distribution function curve is faster. However, when we improve $e(5, 2)$, probability density function curve and cumulative distribution function curve are rarely changed. We also improve the other links respectively, and the result (not plotted) is that the end-to-end delay distribution is rarely improved in each situation.

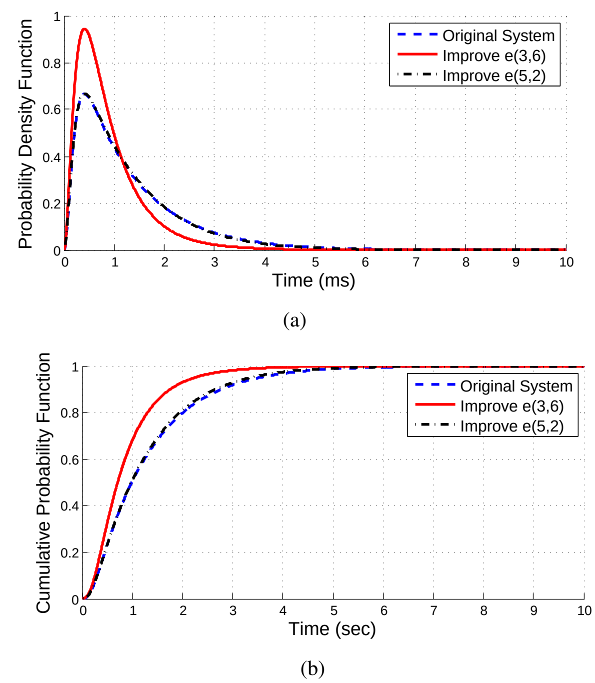

Fig. 3. The end-to-end delay of the example system after improve some links.

References

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[2] J. J. D'Azzo, C.H. Houpis, "Linear control system analysis and design with MATLAB (fifth edition)," 2003.